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The required equation is found by eliminating l, m, n and R giving the equation,

$$\left.\begin{vmatrix} x^2+y^2+z^2 & x & y & z & 1 \\ a_1^2+b_1^2+c_1^2 & a_1 & b_1 & c_1 & (1\pm r_1)^2 \\ a_2^2+b_2^2+c_2^2 & a_2 & b_2 & c_2 & (1\pm r_2)^2 \\ x_1^2+y_1^2+z_1^2 & x_1 & y_1 & z_1 & 1 \\ x_2^2+y_2^2+z_2^2 & x^2 & y^2 & z^2 & 1. \end{vmatrix}\right\} =0.$$

CALCULUS

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

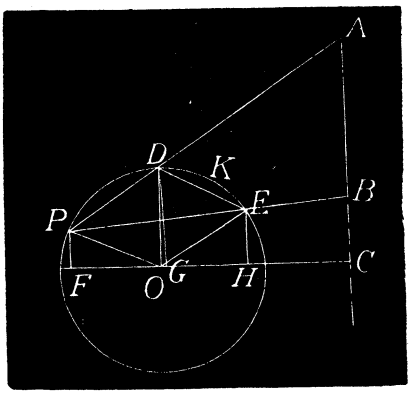
SOLUTIONS TO PROBLEMS.

10. Proposed by ERIC DOOLITTLE, Instructor in Mathematics, State University of Iowa.

Prove or disprove the following theorem: If O be any circle, and AB any straight line either within or without the circumference, and if a perpendicular be dropped from O upon AB and prolonged backward to meet the circumference in P, *then* will the angle whose vertex lies at P and whose sides pass through A and B, cut a portion CPD from the circle which shall be greater than that cut by any other angle whose vertex lies on the circumference and whose sides pass through A and B.

Solution by G. B. M. ZERR, Principal of High School, Staunton, Virginia.

Let O be the centre of the given circle A, B the given points in the line ABC, OC the perpendicular from O on ABC, P any point in the circumference of the circle, D the point where AP cuts the circumference, E the point where PB cuts the circumference.



Let (x_1,y_1) be the coordinates of P, and $(a,b,)$and(a,c) the coordinates of A, and B; O the origin then $x^2+y^2=r^2$ is the equation to the circle, also $x_1^2+y_1^2=r^2$.

Equation to PA is $(a-x_1)y+(y_1-b)x=ay_1-bx_1$ . . . . (2).

Equation to PB is $(a-x_1)y+(y_1-c)x=ay_1-cx_1$ . . . . (3).

Eliminating x between $x^2+y^2=r^2$ and (2) we get $y^2+\dfrac{2y(a-x_1)(bx_1-ay_1)}{(a-x_1)^2+(b-y_1)^2}=\dfrac{r^2(b-y_1)^2-(bx_1-ay_1)^2}{(a-x_1)^2+(b-y_1)^2}$.

$$\therefore\ y+\frac{(a-x_1)(bx_1-ay_1)}{(a-x_1)^2+(b-y)^2}=\pm\frac{(b-y_1)\sqrt{r^2(b-y_2)^2+r^2(a-x_1)^2-(bx_1-ay_1)^2}}{(a-x_1)^2+(b-y_1)^2}$$

but $r^2(b-y_1)^2+r^2(a-x_1)^2=a^2r^2+b^2r^2+r^4-2r^2(ax_1+by_1)$

and $(bx_1-ay_1)^2=-(ax_1+by_1)^2+a^2r^2+b^2r^2$.

$$\therefore\quad y+\frac{(a-x_1)(bx_1-ay_1)}{(a-x_1)^2+(b-y_1)^2}=\pm\frac{(b-y_1)(r^2-ax_1-by_1)}{(a-x_1)^2+(b-y_1)^2}.$$

$$\therefore\ y=\frac{2br^2+(a^2-b^2-r^2)y_1-2abx_1}{(a-x_1)^2+(b-y_1)^2}\quad\text{and } y=y_1.$$

$$x=\frac{2ar^2+(b^2-a^2-r^2)x_1-2aby_1}{(a-x_1)^2+(b-y_1)^2}\quad\text{and } x=x_1.$$

Similarly, $y=\dfrac{2cr^2+(c^2-a^2-r^2)y_1-2acx_1}{(a-x_1)^2+(c-y_1)^2}$ and $y=y_1$.

$$x=\frac{2ar^2+(c^2-a^2-r^2)x_1-2acy_1}{(a-x_1)^2+(c-y_1)^2}\quad\text{and } x=x_1.$$

Let $\angle COP=\theta$, $\angle COE=\phi$, $\angle COD=\psi$.

Area $PDKE$=area triangle PDE+area segment DKE.

Area $=\dfrac{r^2}{2}[\sin\theta\,(\cos\psi-\cos\phi)+\sin\phi\,(\cos\theta-\cos\psi)+\sin\psi\,(\cos\phi-\cos\theta)]+\dfrac{r^2}{2}[(\psi-\phi)-\sin(\psi-\phi)]=\dfrac{r^2}{2}[\sin\theta\cos\psi-\sin\theta\cos\phi+\sin\phi\cos\theta-\sin\psi\cos\theta+\psi-\phi]=\dfrac{r^2}{2}[\sin(\theta-\psi)+\sin(\phi-\theta)+\psi-\phi]=$maximum.

$$\text{Sin }\theta=\frac{y'}{r},\ \cos\theta=-\frac{x'}{r},\ \sin\phi=\frac{2cr^2+(a^2-c^2-r^2)y_1-2acx_1}{r(a-x_1)^2+r(c-y_1)^2},$$

$$\cos\theta=\frac{2ar^2+(c^2-a^2-r^2)x_1-2acy_1}{r(a-x_1)^2+r(c-y_1)^2},\ \sin\psi=\frac{2br^2+(a^2-b^2-r^2)y_1-2abx_1}{r(a-x_1)^2+r(b-y_1)^2},$$

$$\cos\psi=\frac{2ar^2+(b^2-a^2-r^2)x_1-2aby_1}{r(a-x_1)^2+r(b-y_1)^2}.$$

$$\therefore\quad\text{Area}=\frac{r^2}{2}\Bigg[\frac{2ar^2y_1+(b^2-a^2-r^2)x_1y_1-2aby_1^2}{r^2(a-x_1)^2+r^2(b-y_1)^2}$$

$$-\frac{2ar^2y_1+(c^2-a^2-r^2)x_1y_1-2acy_1^2}{r^2(a-x_1)^2+r^2(c-y_1)^2}-\frac{2cr^2x_1+(a^2-c^2-r^2)x_1y_1-2acx_1^2}{r^2(a-x_1)^2+r^2(c-y_1)^2}$$

$$+\frac{2br^2x_1+(a^2-b^2-r^2)x_1y_1-2abx_1^2}{r^2(a-x_1)^2+r^2(b-y_1)^2}+\cos^{-1}\left\{\frac{2ar^2+(b^2-a^2-r^2)x_1-2aby_1}{r(a-x_1)^2+r(b-y_1)^2}\right\}$$

$$-\cos^{-1}\left\{\frac{2ar^2+(c^2-a^2-r^2)x_1-2acy_1}{r(a-x_1)^2+r(c-y_1)^2}\right\}\Bigg].$$

This reduces to

$$\text{Area}=\frac{r^2}{2}\Bigg[\frac{2(a-x_1)(y_1-b)}{(a-x_1)^2+(b-y_1)^2}-\frac{2(a-x_1)(y_1-c)}{(a-x_1)^2+(c-y_1)^2}$$

$$+\cos^{-1}\left\{\frac{2ar^2+(b^2-a^2-r^2)x_1-2aby_1}{r(a-x_1)^2+r(b-y_1)^2}\right\}$$

$$-\cos^{-1}\left\{\frac{2ar^2+c(c^2-a^2-r^2)x_1-2acy_1}{r(a-x_1)^2+r(c-y_1)^2}\right\}\Bigg].$$

Differentrating this expression and also $x_1^2+y_1^2=r^2$ and eliminating dx_1, dy_1, we get

$$2(x_1-y_1)\left\{\frac{a+b-x_1-y_1}{[(a-x_1)^2+(b-y_1)^2]^2}-\frac{a+c-x_1-y_1}{[(a-x_1)^2+(c-y_1)^2]^2}\right\}$$
$$-\frac{2b(r-a)x_1-[b^2-(a-r^2)]y_1}{[r(a-x_1)^2+r(b-y_1)^2][2br^2+(a^2-b^2-r^2)y_1-2abx_1]}$$
$$+\frac{2c(r-a)x_1-[c^2-(a-r)^2]y_1}{[r(a-x_1)^2+r(c-y_1)^2][2cr^2+(a^2-c^2-r^2)y_1-2acx_1]}=0.$$

From this expression the values of x_1, y_1 that give a maximum can be determined.

11. Proposed by H. W. DRAUGHON, Clinton, Louisiana.

A ribbon, 1 inch wide is wrapped spirally around a right prism, altitude 10 ft., bases of n sides, radius of circumscribed circle, 1 ft., so as to cover the entire convex surface. (1) What is the length of the ribbon? (2) If the ribbon is unwound and kept tense, by a power acting on the lower end, and moving in the plane of the lower base, what will be the length of the curve described by the power?

Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let $ABCD\ldots$ be the projection of the base of the prism on the plane of the paper, O its centre, OA the radius of the circumscribed circle $\angle BOA=\frac{2\pi}{n}$.

$\angle ABF=\angle BCG=$&c.$=\frac{2\pi}{n}$, $AB=2r\sin\frac{\pi}{n}$ where $r=OA$.

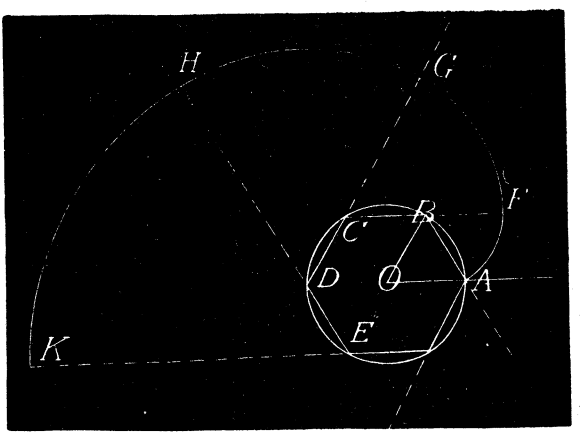
(1.) $\therefore$ perimeter of base of prism $=2rn\sin\frac{\pi}{n}$.

Now since the ribbon crosses each edge at an inch apart, the length for one revolution $=\sqrt{4r^2n^2\sin^2\frac{\pi}{n}+\frac{1}{144}}$ feet.

There are in all 120 revolutions, hence,

length $=120\sqrt{4r^2n^2\sin^2\frac{\pi}{n}+\frac{1}{144}}=10\sqrt{576n^2\sin^2\frac{\pi}{n}+1}$. Since $r=1$, when $n=6$, $l=10\sqrt{(576\times 9+1)}=720.07$ feet, and when $n=$ infinity, $n\sin\frac{\pi}{n}=\pi$.

$\therefore$ $l=10\sqrt{(576\,\pi^2+1)}=754.05$ feet.



(2) Let m be the number of revolutions. Also let $AFGHK\ldots$ be the curve described.

From A to F, $AB=2r\sin\frac{\pi}{n}$ is the radius and $\frac{2\pi}{n}$ the central angle, from F to